Time-Series Constraints

Ekaterina Arafailova, Nicolas Beldiceanu, Mats Carlsson, Rémi Douence, Pierre Flener, M. Andreína Francisco R., Justin Pearson, and Helmut Simonis

8th December 2016

About Me

- Graduated from M2 ORO in 2015.
- Internship at EMN under supervision of N. Beldiceanu (EMN) and M. Minoux (LIP6).
- Started a PhD on the 1st of September, 2015.
- Thesis Director: N. Beldiceanu, Co-Supervisor: R. Douence.
- GRACeFUL project.

Collaboration

1. GRACeFUL people.
2. Helmut Simonis (University College Cork).
3. Pierre Flener, M. Andreína Francisco R., Justin Pearson (The University of Uppsala).
4. Mats Carlsson (SICS)

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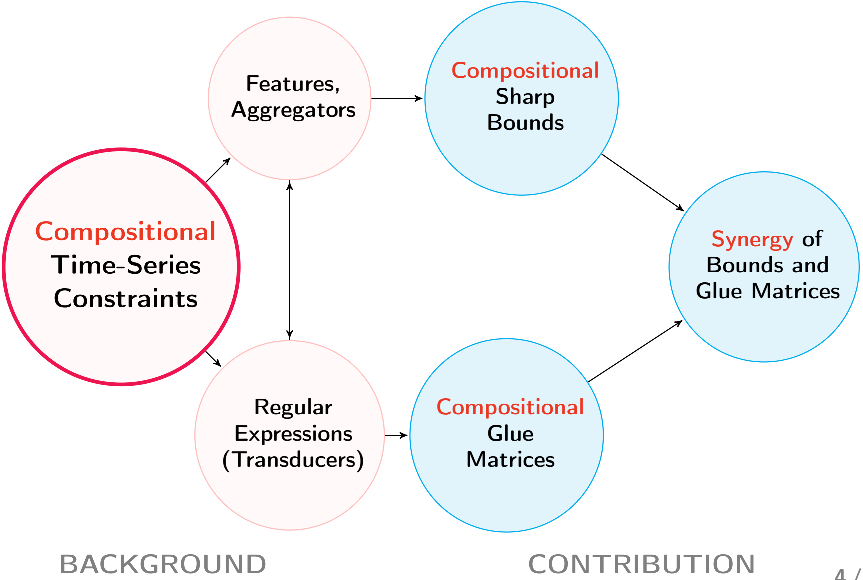
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Aggregators
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Sharp
Bounds
Compositional
Time-Series
Constraints
Synergy of
Bounds and
Glue Matrices
Regular
Expressions
(Transducers)
Compositional
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BACKGROUND
CONTRIBUTION

Example of a Time-Series Constraint

Constrain the maximum of the minima of the valleys in the time series $\langle 5, 5, 6, 4, 6, 6, 4, 2, 4, 4, 1, 1, 1, 1, 3, 0 \rangle$.

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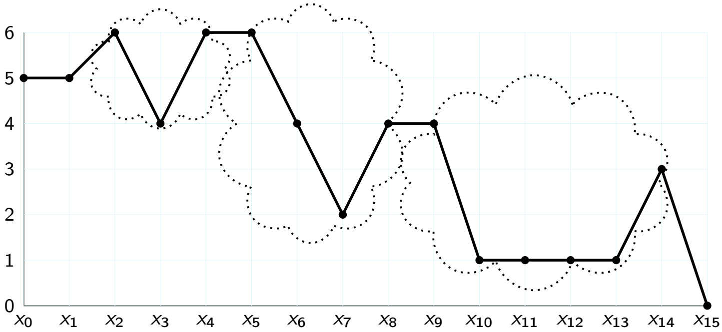

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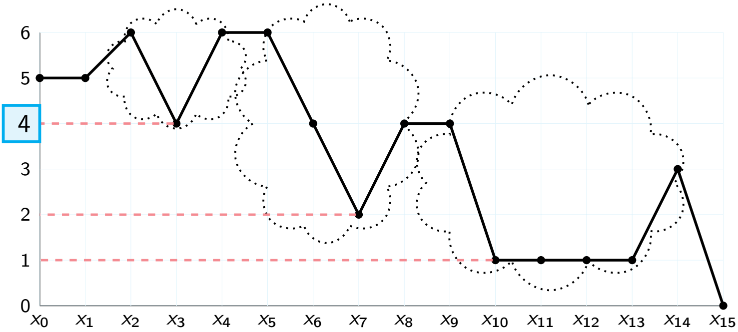

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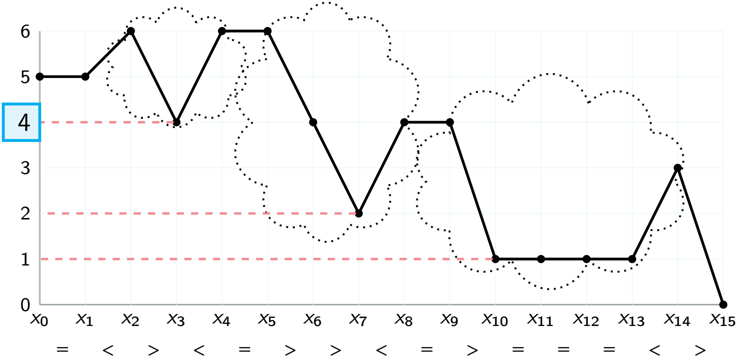

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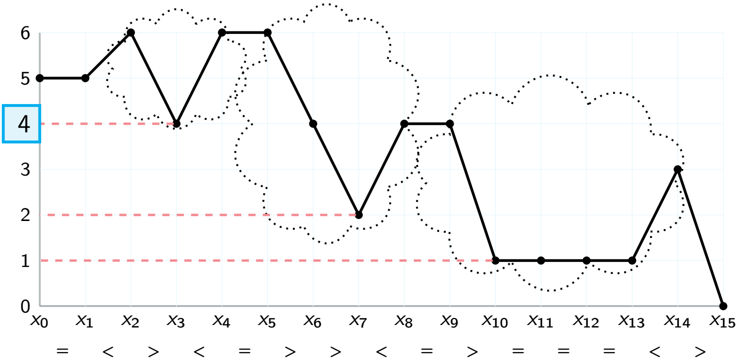

A subsequence $\langle X_i, \ldots, X_j \rangle$ of $\langle X_0, \ldots, X_m \rangle$ is a **valley** if the signature of $\langle X_{i-1}, \ldots, X_{j+1} \rangle$ is a maximal word matching **'>(>|=)*(<|=)*<'**.

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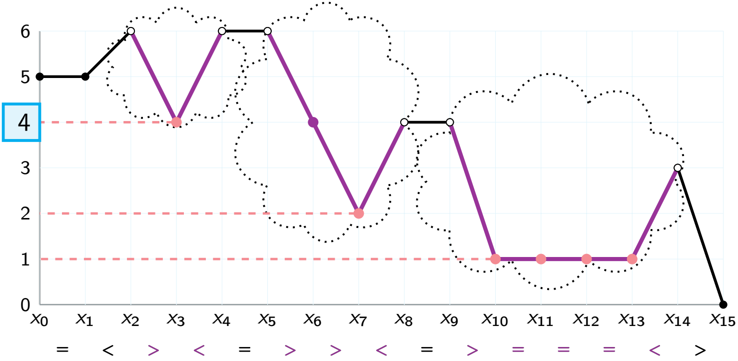

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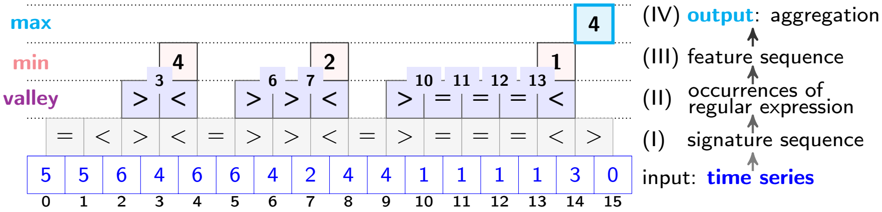
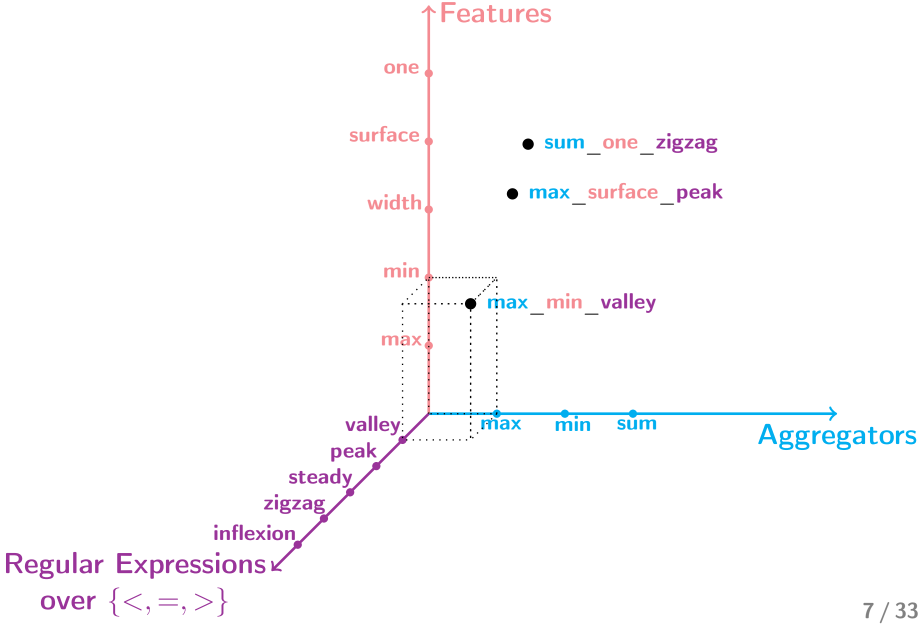
Compositional Time-Series Definition by Multiple Layers of Functions



max_min_valley(⟨5, 5, 6, 4, 6, 6, 4, 2, 4, 4, 1, 1, 1, 1, 3, 0⟩, 4) holds

Space of Time-Series Constraints



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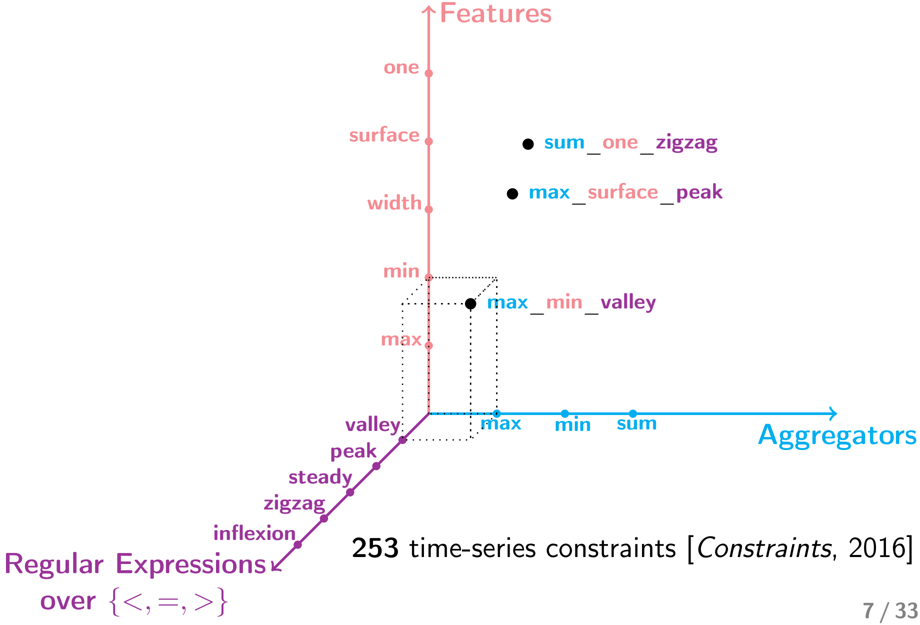

253 time-series constraints [*Constraints*, 2016]

Main Question

**How to go from a compositional constraint definition
to compositional combinatorial objects
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to compositional combinatorial objects
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Time-series constraints are very numerous:
we cannot afford to consider each of them separately.

Applications of Time-Series Constraints

- Analysis of output of electric power stations over multiple days. [*CP 2013*]
- Power management for large-scale-distributed systems. [*Computing*, 2016]
- Trace analysis for Internet Service Provider. (CeADAR)
- Anomaly detection and error correction in building data. (Campus21)

Synthesising Time-Series Constraints

Use a transducer: [*Constraints*, 2016]

- Input sequence: Handle the **matching** aspect
- Output sequence: Handle the **computational** aspect

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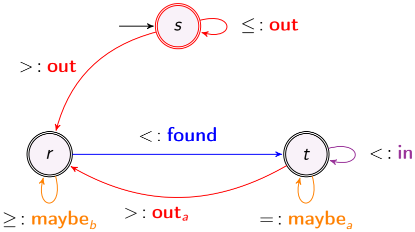

Transducer for **valley**

Synthesising Time-Series Constraints

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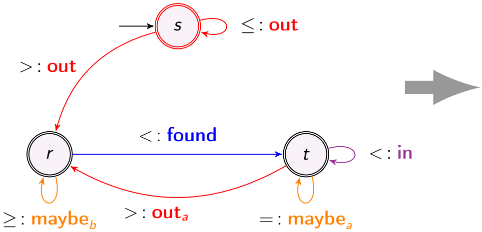

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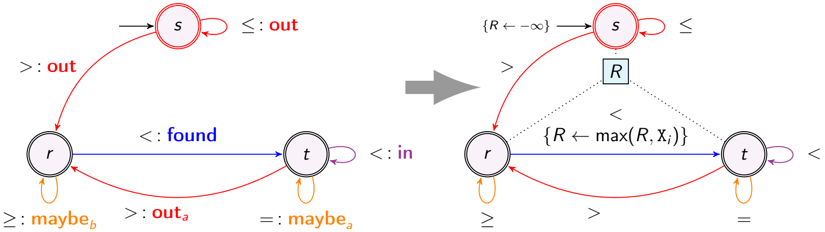

Transducer for **valley** Automaton for **max_min_valley**

Synthesising Time-Series Constraints

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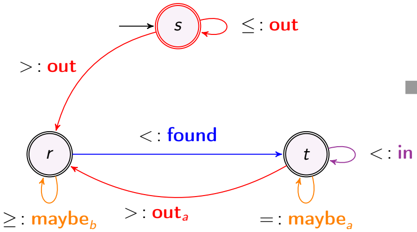
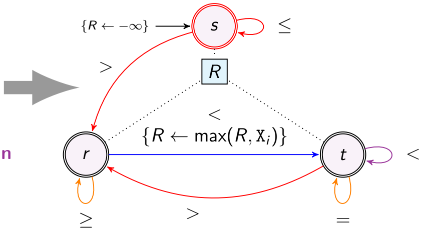


Transducer for **valley**



Automaton for **max_min_valley**

5	$\xrightarrow{=}$	5	$\xrightarrow{<}$	6	$\xrightarrow{>}$	4	$\xrightarrow{<}$	6	$\xrightarrow{=}$	6	$\xrightarrow{>}$	4	$\xrightarrow{>}$	2	$\xrightarrow{<}$	4
s	out	s	out	s	out	r	found	t	maybe$_a$	t	out$_a$	r	maybe$_b$	r	found	t

Implementing Time-Series Constraints

- CP: **Reformulation of automata** with accumulators*
- MIP: **Linear reformulation** of automata with accumulators**

* [*Constraints*, 2005], ** [*CPAIOR 2016*]

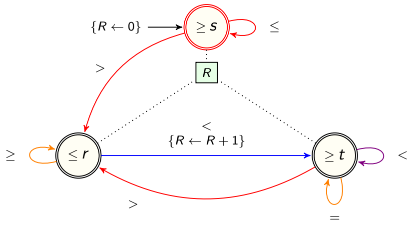
Goal: Address the combinatorial aspect of time-series constraints.

Method: Exploit the **compositional nature** of time-series constraints at the combinatorial level.

Input

- Time-series variables X_i with i in $[1, n]$ over their domains $[a_i, b_i]$
- An automaton with accumulators for a time-series constraint with
  - a set of states Q;
  - an input alphabet Σ;
  - an m-tuple of integer accumulators with their initial values $I = \langle I_1, \ldots, I_m \rangle$;
  - a transition function $\delta : Q \times Z^m \times \Sigma \to Q \times Z^m$.

Input: example



Automaton for nb_valley

$$\frac{\mathbf{5}}{s} \xrightarrow{=} \frac{\mathbf{5}}{s} \xrightarrow{<} \frac{\mathbf{6}}{s} \xrightarrow{>} \frac{\mathbf{4}}{r} \xrightarrow{<} \frac{\mathbf{6}}{t} \xrightarrow{=} \frac{\mathbf{6}}{t} \xrightarrow{>} \frac{\mathbf{4}}{r} \xrightarrow{>} \frac{\mathbf{2}}{r} \xrightarrow{<} \frac{\mathbf{4}}{t}$$

$$\frac{\mathbf{5}}{0} \xrightarrow{=} \frac{\mathbf{5}}{0} \xrightarrow{<} \frac{\mathbf{6}}{0} \xrightarrow{>} \frac{\mathbf{4}}{0} \xrightarrow{<} \frac{\mathbf{6}}{1} \xrightarrow{=} \frac{\mathbf{6}}{1} \xrightarrow{>} \frac{\mathbf{4}}{1} \xrightarrow{>} \frac{\mathbf{2}}{1} \xrightarrow{<} \frac{\mathbf{4}}{2}$$

Goal

Our Goal

A way to generate a model for an automaton with linear or linearisable accumulator updates, for example containing min and max.

Linear decomposition of automata without accumulators

Côté, M.C., Gendron, B., Rousseau, L.M.: Modeling the regular constraint with integer programming. In: CPAIOR 2007. LNCS, vol. 4510, pp. 29–43. Springer (2007)

Signature Constraints

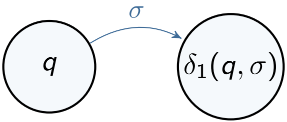
Introduced variables: S_i over Σ with $i \in [0, n-2]$.

What do the values of S_i mean ?

$$S_i = \text{'}{>}\text{'} \Leftrightarrow X_i > X_{i+1}, \forall i \in [0, n-2]$$
$$S_i = \text{'}{=}\text{'} \Leftrightarrow X_i = X_{i+1}, \forall i \in [0, n-2]$$
$$S_i = \text{'}{<}\text{'} \Leftrightarrow X_i < X_{i+1}, \forall i \in [0, n-2]$$

Transition Function Constraints

Introduced variables: Q_i over Q with $i \in [0, n-1]$; T_i over $Q \times \Sigma$ with $i \in [0, n-2]$



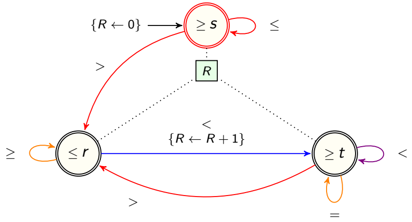
Each transition constraint has a form:

$Q_i = q \wedge S_i = \sigma \Leftrightarrow Q_{i+1} = \delta_1(q, \sigma) \wedge T_i = \langle q, \sigma \rangle,$
$\forall i \in [0, n-2],\ \forall q \in Q,\ \forall \sigma \in \Sigma$

Initial state is fixed:

$Q_0 = q_0$

Accumulator Updates



Accumulator updates

R_i over $[a, b]$ with i in $[0, n-1]$; T_i over $Q \times \Sigma$ with i in $[0, n-2]$.

- $R_0 = 0$
- $T_i = \langle r, > \rangle \Rightarrow \mathbf{R_{i+1}} = \mathbf{R_i} + \mathbf{1}, \forall i \in [0, n-2]$
- $T_i = \langle q, \sigma \rangle \Rightarrow \mathbf{R_{i+1}} = \mathbf{R_i}, \forall i \in [0, n-2], \forall \langle q, \sigma \rangle \in (Q \times \Sigma) \setminus \langle r, > \rangle$

New Variables for the Linear Model

New variables

- Q_i is replaced by 0-1 variables Q_i^q for all q in Q.
  $Q_i^q = 1 \Leftrightarrow Q_i = q$
- New constraint: $\sum_{q \in Q} Q_i^q = 1, \forall i \in [0, \ldots, n-1]$
- The same procedure for T_i and S_i wrt their domains
- X_i and R_i remain integer variables!

- Every constraint of the logical model is made linear by applying some standard techniques
- The linear model has $O(n)$ variables and $O(n)$ constraints

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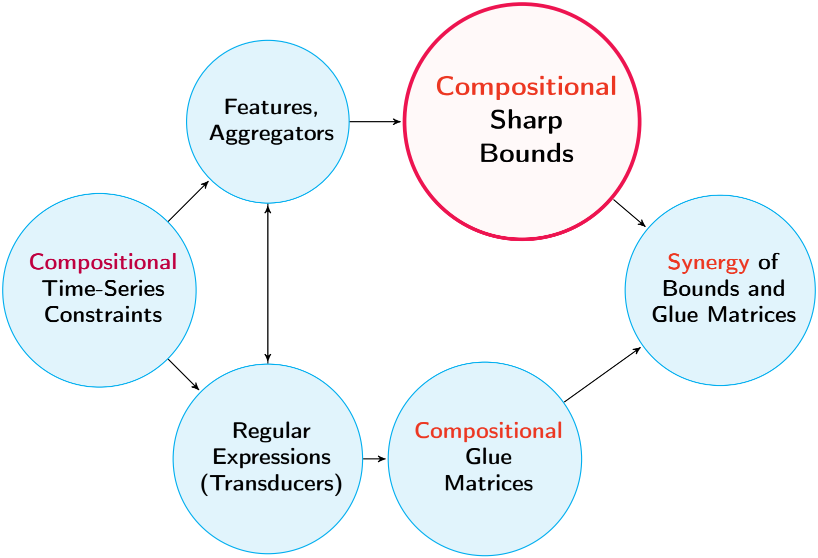
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Compositional Sharp Bounds
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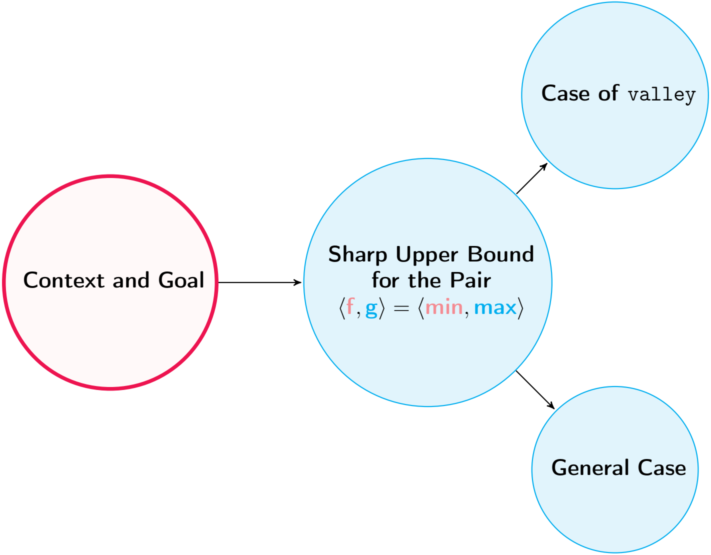
Compositional Sharp Bounds: Contents



Context and Goal

Consider a $\mathtt{g}\_\mathtt{f}\_\sigma(\langle X_0, \dots, X_m \rangle, N)$ time-series constraint, for a regular expression σ, a feature $\mathtt{f}$, and an aggregator $\mathtt{g}$, with every X_i ranging over **the same** integer interval $[a, b]$.

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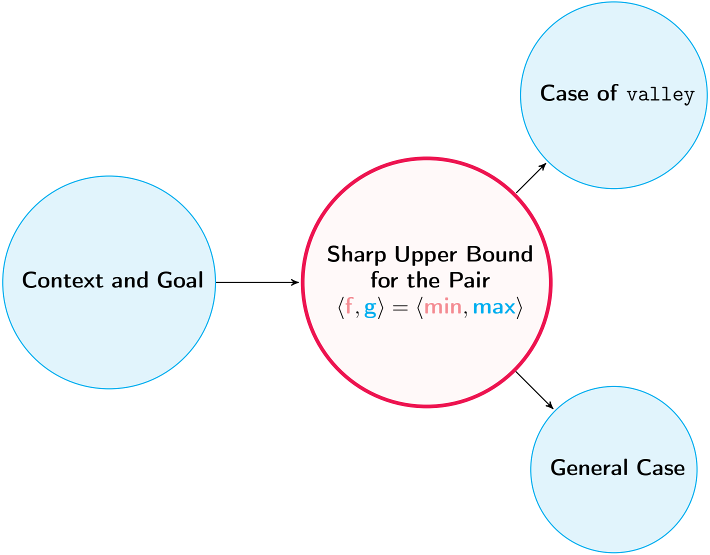
Goal: Derive formulae $\beta^{\mathbf{u}}_{\mathtt{f},\mathtt{g}}$ and $\beta^{\ell}_{\mathtt{f},\mathtt{g}}$ that are **parametrised by σ** and yield respectively sharp upper and lower bounds on the value of N.

Compositional Sharp Bounds: Contents



Computing a Sharp Upper Bound for the pair $\langle \mathbf{min}, \mathbf{max} \rangle$

Consider a $\mathbf{max\_min\_}\sigma(\langle X_0, \ldots, X_m \rangle, N)$ time-series constraint with every X_i ranging over $[a, b]$.

Goal: Compute a sharp upper bound on the value of N.

Computing a Sharp Upper Bound for the pair $\langle \mathbf{min}, \mathbf{max} \rangle$

Consider a max_min_$\sigma(\langle X_0, \ldots, X_m \rangle, N)$ time-series constraint with every X_i ranging over $[a, b]$.

Goal: Compute a sharp upper bound on the value of N.

Method:

1. Introduce the **characteristics** of a regular expression σ: *the minimal difference between the domain upper bound b and the minima of the subseries*, which is called the **shift** of σ and denoted by Δ_σ.

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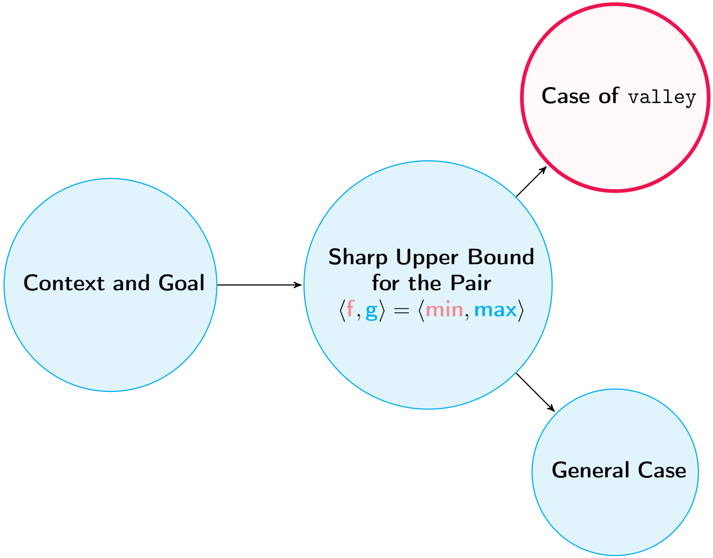
Method:

1. Introduce the **characteristics** of a regular expression σ: *the minimal difference between the domain upper bound b and the minima of the subseries*, which is called the **shift** of σ and denoted by Δ_σ.
2. Explore the connection between this characteristics and the sharp upper bound on the value of N.

Compositional Sharp Bounds: Contents



Computing a Sharp Upper Bound for max_min_valley

Consider the max_min_valley$(\langle X_0, \ldots, X_8 \rangle, N)$ time-series constraints with every X_i ranging over $[a, b] = [3, 6]$.

Shift of valley and Its Connection to a Sharp Upper Bound

- The shift of valley is the smallest possible difference between b and the minima of valleys, namely 1 and is reached, for example, for valleys whose signature is ‘$><$’.

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- The minimum of any valley is at least $\Delta_{\texttt{valley}}$, which is 1, away from b, which is 6, thus $N \leq b - \Delta_{\texttt{valley}} = 6 - 1 = 5$.

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- This bound is reached for some ground time series.

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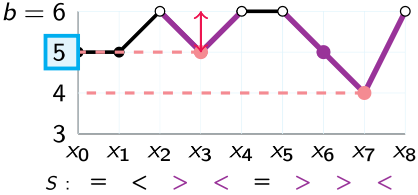
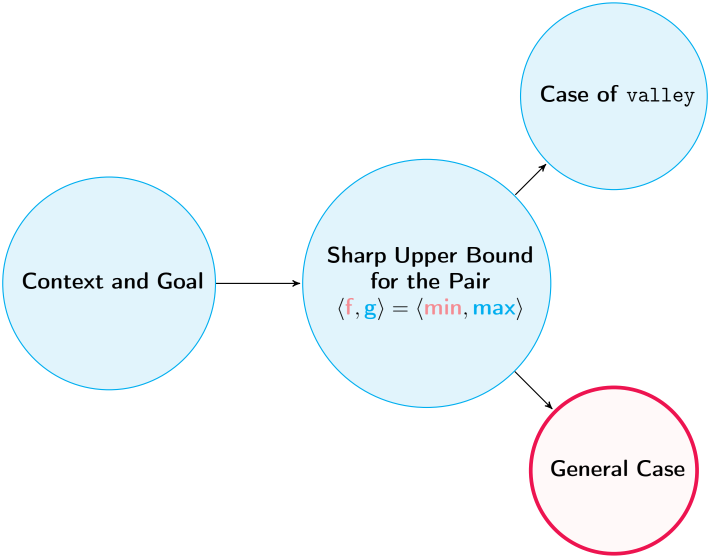
$$N = b - \Delta_{\texttt{valley}} = 6 - 1 = 5$$



Compositional Sharp Bounds: Contents



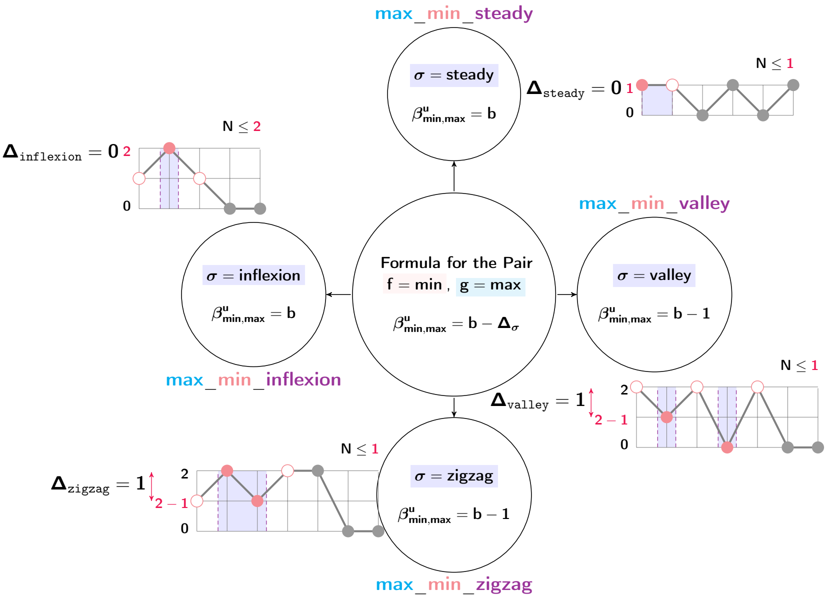
Sharp Upper Bound for the General Case

Theorem

Consider a max_min_$\sigma(\langle X_0, \ldots, X_m \rangle, N)$ time-series constraint, where σ is one of the 22 regular expressions and every X_i ranges over $[a, b]$, then the sharp upper bound on the value of N is

$$\beta^{\mathbf{u}}_{\mathtt{min},\mathtt{max}} = b - \Delta_{\sigma}$$

Compositional Bounds (Summary)



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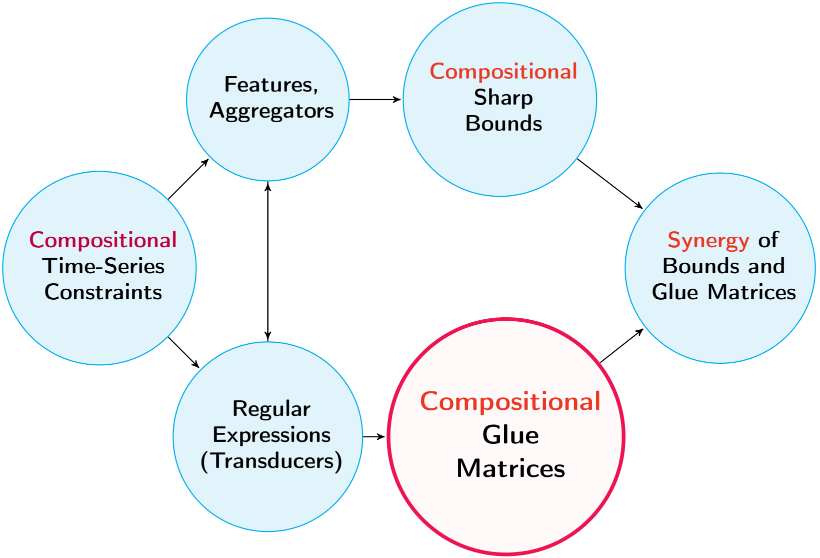
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Linking the Prefixes and Suffixes of a Sequence

For the ngroup$(\langle X_0, \ldots, X_m \rangle, \mathcal{S}, N)$ constraint, N is the number of maximal subsequences of $\langle X_0, \ldots, X_m \rangle$ whose values are all in $\mathcal{S}$.

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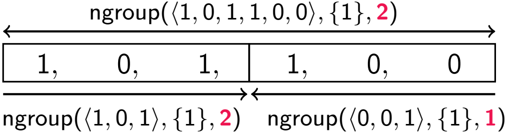

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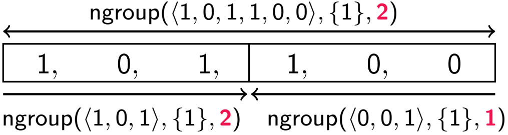

not just a sum ($2 \neq 2 + 1$)...

Linking the Prefixes and Suffixes of a Sequence

For the $\text{ngroup}(\langle X_0, \ldots, X_m \rangle, \mathcal{S}, N)$ constraint, N is the number of maximal subsequences of $\langle X_0, \ldots, X_m \rangle$ whose values are all in $\mathcal{S}$.

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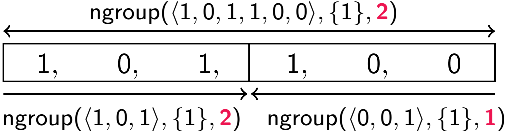
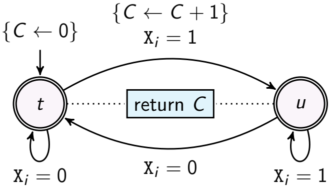


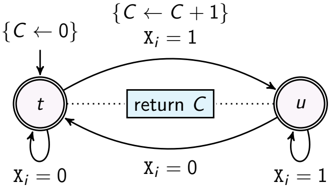
not just a sum $(2 \neq 2 + 1)$...how to compute the correction term -1?

Glue Matrix for the ngroup Constraint



Automaton for **ngroup**

Glue Matrix for the ngroup Constraint



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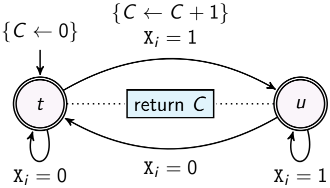
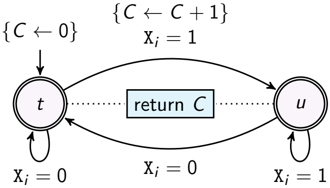


Automaton for **ngroup**

	t	**u**
t	0	0
u	0	-1

Glue Matrix for **ngroup**

Glue Matrix for the ngroup Constraint



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	t	**u**
t	0	0
u	0	**−1**

Glue Matrix for **ngroup**

ngroup$(\langle 1,0,1,1,0,0\rangle, \{1\}, 2)$

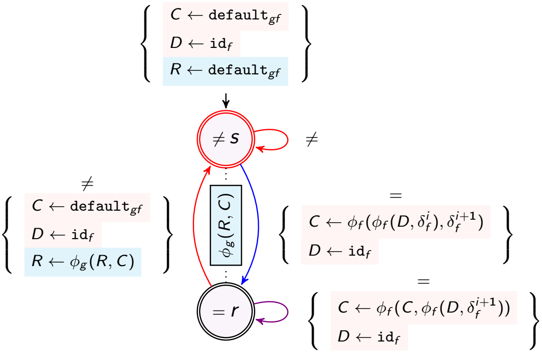
	1,	0,	1,		1,	0,	0
t,	u,	t,	**u**,	**u**,	t,	t,	t

ngroup$(\langle 1,0,1\rangle, \{1\}, 2)$ ngroup$(\langle 0,0,1\rangle, \{1\}, 1)$

Compositional Glue Matrices for Time-Series Constraints

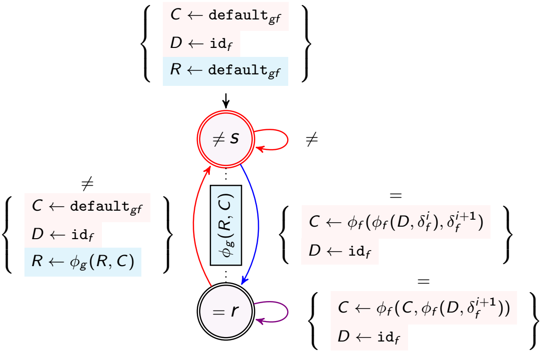
Goal: For a transducer, derive a glue matrix
that is parametrised by a feature and an aggregator.

Generic Glue Matrix for steady_sequence = '$=^+$'



Generic Automaton for
g_f_steady_sequence

Generic Glue Matrix for steady_sequence = '$=^+$'

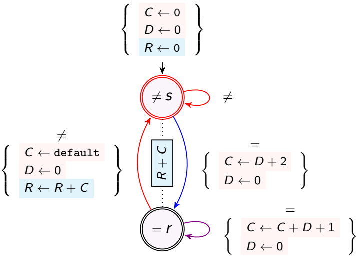


Generic Automaton for g_f_steady_sequence

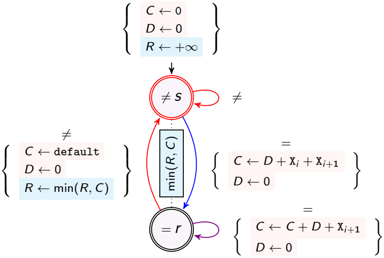
	s	r
s	$\phi_g(\overrightarrow{C}, \overleftarrow{C})$	$\phi_g(\overrightarrow{C}, \overleftarrow{C})$
r	$\phi_g(\overrightarrow{C}, \overleftarrow{C})$	$\phi_f(\overrightarrow{C}, \overleftarrow{C}, \overrightarrow{D}, \overleftarrow{D}, \delta_f^i)$

Generic Glue Matrix for g_f_steady_sequence

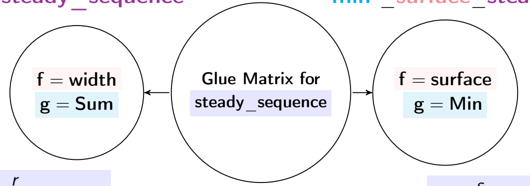
Corresponding Compositional Glue Matrices



sum _width_steady_sequence



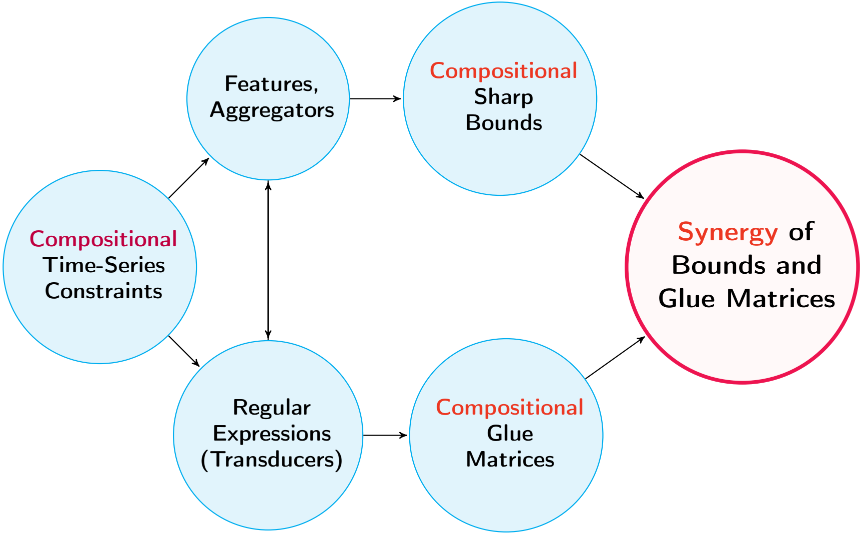
min _surface_steady_sequence



	s	r
s	$\overrightarrow{C} + \overleftarrow{C}$	$\overrightarrow{C} + \overleftarrow{C}$
r	$\overrightarrow{C} + \overleftarrow{C}$	$\overrightarrow{C} + \overleftarrow{C} + \overrightarrow{D} + \overleftarrow{D} + 1$

	s	r
s	$\min(\overrightarrow{C}, \overleftarrow{C})$	$\min(\overrightarrow{C}, \overleftarrow{C})$
r	$\min(\overrightarrow{C}, \overleftarrow{C})$	$\overrightarrow{C} + \overleftarrow{C} + \overrightarrow{D} + \overleftarrow{D} + X_i$

Contents



Features, Aggregators
Compositional Sharp Bounds
Compositional Time-Series Constraints
Synergy of Bounds and Glue Matrices
Regular Expressions (Transducers)
Compositional Glue Matrices

Synergy of Sharp Bounds and Glue Matrices

Consider a g_f_$\sigma(\langle X_0, \ldots, X_m \rangle, N)$ time-series constraint where all X_i range over the same domain.

- Add **sharp bounds** for every prefix and every reversed suffix of $X_0, \ldots, X_m$.
- Impose **glue-matrix implied constraints** on every prefix and every reversed suffix of $X_0, \ldots, X_m$.

The combination of sharp bounds and glue matrices improves propagation.

Conclusion: clear separation between three levels

- **Language**: compositional description of constraints with aggregator g, feature f, regular expression σ

- Abstract Combinatorial Objects (*transducers and formulae*)
  - Transducers for each regular expression σ (22)
  - Parametrised bounds for each pair g,f (20)
  - Parametrised glue matrices for each regular expression σ (22)

MANUAL

- Multiple usages (*synthesised code:* LaTeX(154100), Prolog (135267))
  - **CP**: enhanced propagation (*bounds, glue matrices*)
  - **Local search**: constant time probing (*glue matrices*)
  - **Data mining**: range of variation of feature (*bounds*)

SYNTHESISED

**Separate definition from usage,
be compositional at all levels**

Publications on Time-Series Constraints

- N. Beldiceanu, R. Douence, M. Carlsson, H. Simonis. "Using Finite Transducers for Describing and Synthesising Structural Time-Series Constraints". Constraints'16.
- E. Arafailova, N. Beldiceanu, R. Douence, M. A. F. Rodriguez, P. Flener, J. Pearson, H. Simonis. "Time-series constraints: Improvements and application in CP and MIP contexts". CPAIOR'16.
- E. Arafailova, N. Beldiceanu, M. Carlsson, M. A. F. Rodriguez, P. Flener, J. Pearson, H. Simonis. "Systematic Derivation of Bounds and Glue Constraints for Time-Series Constraints". CP'16.
- E. Arafailova, N. Beldiceanu, R. Douence, M. Carlsson, M. A. F. Rodriguez, P. Flener, J. Pearson, H. Simonis. "Global Constraint Catalog, Volume II, Time-Series Constraints". CoRR.

Arafailova, Ekaterina et al. (2016). "Time-series constraints: Improvements and application in CP and MIP contexts". In: *CPAIOR 2016*. Ed. by Claude-Guy Quimper. Vol. 9676. LNCS. Springer, pp. 18–34.

Beldiceanu, Nicolas et al. (2005). "Reformulation of global constraints based on constraints checkers". In: *Constraints* 10.4, pp. 339–362.

Beldiceanu, Nicolas et al. (2013). "Describing and generating solutions for the EDF unit commitment problem with the ModelSeeker". In: *CP 2013*. Ed. by Christian Schulte. Vol. 8124. LNCS. Springer, pp. 733–748.

Beldiceanu, Nicolas et al. (2016a). "Towards energy-proportional Clouds partially powered by renewable energy". In: *Computing*, p. 20. url: https://hal.inria.fr/hal-01340318.

Beldiceanu, Nicolas et al. (2016b). "Using finite transducers for describing and synthesising structural time-series constraints". In: *Constraints* 21.1, pp. 22–40.